Trigonometry 4

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Q> Find the range of $\sec^2\theta + \cos^2\theta$ for $\theta \in [0°, 360°]$

Q> Find the range of $\sec\theta + \cos\theta$ for $\theta \in [0°, 360°]$

Q> Find the range of $\sqrt{1+\tan^2\theta} + \cos\theta$ for $\theta \in [0°, 360°]$

Q> Find the range of $\sqrt{1+\tan^2\theta} + \sqrt{1-\sin^2\theta}$ for $\theta \in [0°, 360°]$

Q> Find the range of $\tan\theta \sec\theta$ for $\theta \in [0°, 360°]$

$$\sec^2\theta + \cos^2\theta = \cos^2\theta + \frac{1}{\cos^2\theta} = \Big(\underbrace{\cos\theta - \frac{1}{\cos\theta}}_{}\Big)^2 + 2 \geq 2$$

$\downarrow$ $[2, \infty)$

0 when $\theta = 0$

∞ when $\theta \to 90°$

$$\sec\theta + \cos\theta = \cos\theta + \frac{1}{\cos\theta}$$

$\xrightarrow{\cos\theta > 0}$ $\left(\sqrt{\cos\theta} - \frac{1}{\sqrt{\cos\theta}}\right)^2 + 2 \in [2, \infty)$

$\xrightarrow{\cos\theta < 0}$ $-\left((-\cos\theta) + \frac{1}{(-\cos\theta)}\right)$

$= -\left(\sqrt{-\cos\theta} - \frac{1}{\sqrt{-\cos\theta}}\right)^2 - 2 \in (-\infty, -2]$

$\downarrow$ $(-\infty, -2] \cup [2, \infty)$

$$\sqrt{1+\tan^2\theta} = |\sec\theta| \qquad \sqrt{x^2} = |x|$$

$y^2 = x^2$

$y = \pm x = \pm\sqrt{y}$

$\sqrt{y} = |y|$

$$\sqrt{\left(\frac{1}{-\cos\theta}\right)^2} = \frac{1}{-\cos\theta} = |\sec\theta|$$

> 0

$\sqrt{(-2)^2} = \sqrt{4} = 2$

$\cancel{-\sqrt{4}}$

$$\sqrt{1+\tan^2\theta} + \cos\theta = |\sec\theta| + \cos\theta$$

$\xrightarrow{\cos\theta > 0}$ $[2, \infty)$

$\xrightarrow{\cos\theta < 0}$ $-\frac{1}{\cos\theta} + \cos\theta \in [$

$= \cos\theta - \frac{1}{\cos\theta} = \frac{\cos^2\theta - 1}{\cos\theta} = \frac{-\sin^2\theta}{\cos\theta} \geq 0$

$\theta \to 270° \Rightarrow$ value $\to \infty$

$\downarrow$ $[0, \infty) \cup [2, \infty) = [0, \infty)$

$$\sqrt{1+\tan^2\theta}+\sqrt{1-\sin^2\theta} = |\sec\theta| + |\cos\theta| \xrightarrow{\cos\theta > 0} \cos\theta + \frac{1}{\cos\theta} \in [2,\infty)$$

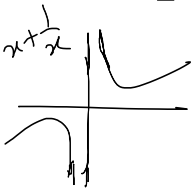
$$\xrightarrow{\cos\theta < 0} -\left(\cos\theta + \frac{1}{\cos\theta}\right) \in [2,\infty)$$

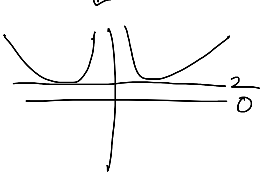
$$\left(\sqrt{-\cos\theta} - \sqrt{\frac{-1}{\cos\theta}}\right)^2 + 2$$

$[2,\infty)$

$|x| + \left|\frac{1}{x}\right|$

$x+\frac{1}{x}$





$$\tan\theta\sec\theta = \frac{\sin\theta}{\cos^2\theta} \xrightarrow{\sin\theta>0} [0,\infty)$$

$$\xrightarrow{\sin\theta<0} (-\infty, 0]$$

$(-\infty,\infty)$

Q) Why did we treat this functions as continuous over $\theta \in [0^\circ, 360^\circ]$?

Ans:- Because trigonometric functions has polynomial expansions, that is,

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} \cdots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n\, \theta^{2n+1}}{(2n+1)!}$$

$$\cos\theta = \sum_{n=0}^{\infty} \frac{(-1)^n\, \theta^{2n}}{(2n)!} = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots$$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ so it is continuous in the part where $\cos\theta \neq 0$